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# An approach to predictive control of multivariable time-delayed plant: Stability and design issues

I. Škrjanc,[a,]* S. Blažič,[a] S. Oblak,[a] J. Richalet[b]

[a]*Faculty of Electrical Engineering, University of Ljubljana, Tržaška 25, Ljubljana, Slovenia*
[b]*Adersa, 10, rue de la Croix Martre, Palaiseau Cedex, France*

**Abstract**

In this paper, a new method of multivariable predictive control is presented. The main advantage of a predictive approach is that multivariable plants with time delays can be easily handled. The proposed control algorithm also introduces a compact and simple design in the case of higher-order and nonminimal phase plants, but it is limited to open-loop stable plants. The algorithm of the proposed multivariable predictive control is developed, designed, and implemented on an air-conditioned system. The stability of the proposed control law is discussed. © 2004 ISA—The Instrumentation, Systems, and Automation Society.

*Keywords:* Time-delayed system; Multivariable control; Predictive control; Thermodynamical plant; Internal model

## 1. Introduction

Predictive control has become a very important area of research in recent years [1]. The principle is based on the forecast of the output signal at each sampling instant. The forecast is done implicitly or explicitly based on a model of the controlled process. In the next step the control is selected which brings the predicted process output signal back to the reference signal in a way that minimizes the difference between the reference and the output signal. The fundamental methods are essentially based on the principle of predictive control by Clarke (generalized predictive control [2,3]), Richalet (model algorithmic control and predictive functional control [4]), Cutler (dynamic matrix control [5]), De Keyser (extended prediction self-adaptive control [6]) and Ydstie (extended horizon adaptive control [7]).

Most industrial plants exhibit a multivariable nature. This means that there are many variables that have to be controlled (usually referred to as outputs). In some cases, a change in one of the manipulated variables mainly affects the corresponding controlled variable and each of the input-output pairs can be considered as a single-input single-output (SISO) plant. In such cases, satisfactory results are usually obtained if the plant is controlled by independent loops. But, in many cases, more than one input variable is coupled with the outputs. When the interactions are not negligible, the plant must be considered as truly multivariable and some type of multivariable control has to be applied to achieve satisfactory performance and/or robustness of the closed-loop system. Multivariable process control has been extensively studied in the literature [8–11]. An important approach to control multivariable processes is to design decoupling compensators to suppress or diminish the interactions and then design common multiple univariable controllers [12–14]. In comparison with classical approaches

*Tel.: +386 1 4768311; fax: +386 1 4264631. *E-mail address*: igor.skrjanc@fe.uni-lj.si

the main advantage of multivariable predictive control is in its simple design and high quality control performance.

In the paper, a new method of multivariable predictive control based on predictive functional control is presented [4]. The algorithm is developed in a state-space domain, which is suitable for dealing with multivariable systems. Tests of the proposed algorithm have been made on an air conditioning system that exhibits a strong interaction between inputs and outputs.

The paper is organized in the following manner: Section 2 deals with the concept of multivariable predictive functional control (MPFC). In Section 3, the mathematical modeling is given and, in Section 4, the implementation of the proposed control algorithm is realized on an air conditioning system. Conclusions are presented in Section 5.

## 2. MPFC control law

In our approach, the basic principles of predictive functional control are applied and extended to a multivariable case. In this instance, the prediction of the plant output is given by its model in the state-space domain. The fundamental principles of predictive functional control [15] are very easy to understand.

The controlled system is multivariable plant with $m$ inputs and $m$ outputs and different delays of individual input-output pairs. Since the control is realized by means of a digital computer, sampled-data models will be used in the paper. The plant can be modelled by a transfer function matrix given by

$$\mathbf{y}_p(k)=\mathbf{G}_p(q)\cdot\mathbf{u}(k)$$
$$=\begin{bmatrix} g_{p_{11}}q^{-T_{11}} & g_{p_{12}}q^{-T_{12}} & \cdots & g_{p_{1m}}q^{-T_{1m}} \\ g_{p_{21}}q^{-T_{21}} & g_{p_{22}}q^{-T_{22}} & \cdots & g_{p_{2m}}q^{-T_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ g_{p_{m1}}q^{-T_{m1}} & g_{p_{m2}}q^{-T_{m2}} & \cdots & g_{p_{mm}}q^{-T_{mm}} \end{bmatrix}\cdot\mathbf{u}(k), \tag{1}$$

where $\mathbf{u}(k)$ and $\mathbf{y}_p(k)$ are vectors of the plant's inputs and outputs, respectively, while $q$ is a shift operator. The integer parameters $T_{ij}$ define the delays of the individual transfer functions. The exact transfer function is not known. However, the identified model is known and can be described by

$$\mathbf{y}_m(k)=\mathbf{G}_m(q)\cdot\mathbf{u}(k)$$
$$=\begin{bmatrix} g_{m_{11}}q^{-\tau_{11}} & g_{m_{12}}q^{-\tau_{12}} & \cdots & g_{m_{1m}}q^{-\tau_{1m}} \\ g_{m_{21}}q^{-\tau_{21}} & g_{m_{22}}q^{-\tau_{22}} & \cdots & g_{m_{2m}}q^{-\tau_{2m}} \\ \vdots & \vdots & \ddots & \vdots \\ g_{m_{m1}}q^{-\tau_{m1}} & g_{m_{m2}}q^{-\tau_{m2}} & \cdots & g_{m_{mm}}q^{-\tau_{mm}} \end{bmatrix}\cdot\mathbf{u}(k) \tag{2}$$

with $\mathbf{y}_m(k)$ the vector of the outputs of the model.

The problem of delays in the plant is circumvented by constructing an auxiliary variable that serves as the output of the plant if there were no delays present. The so-called *undelayed* model of the plant will be introduced for that purpose. It is obtained by *removing* delays from the *delayed* model (2) and converting it to the state-space description:

$$\begin{aligned} \mathbf{x}_m(k+1)&=\mathbf{A}_m\mathbf{x}_m(k)+\mathbf{B}_m\mathbf{u}(k),\\ \mathbf{y}_m^0(k)&=\mathbf{C}_m\mathbf{x}_m(k), \end{aligned} \tag{3}$$

where $\mathbf{y}_m^0(k)$ represents the *undelayed* output of the plant.

The behavior of the closed-loop system is defined by a reference trajectory, which is given in the form of the reference model. The control goal is to determine the future control action so that the predicted output trajectory coincides with the reference trajectory. The coincidence point is called a coincidence horizon and denoted by $H$. The prediction is calculated under the assumption of constant future manipulated variables $[\mathbf{u}(k)=\mathbf{u}(k+1)=\cdots=\mathbf{u}(k+H-1)]$. This strategy is known as mean level control. The $H$-step ahead prediction of the *undelayed* plant output is then obtained from Eq. (3):

$$\begin{aligned} \mathbf{y}_m^0(k+H)=\mathbf{C}_m[&\mathbf{A}_m^H\mathbf{x}_m(k)+(\mathbf{A}_m^H-\mathbf{I})\\ &\times(\mathbf{A}_m-\mathbf{I})^{-1}\mathbf{B}_m\mathbf{u}(k)]. \end{aligned} \tag{4}$$

The reference model is given by the following difference equation:

$$\begin{aligned} \mathbf{x}_r(k+1)&=\mathbf{A}_r\mathbf{x}_r(k)+\mathbf{B}_r\mathbf{w}(k),\\ \mathbf{y}_r(k)&=\mathbf{C}_r\mathbf{x}_r(k), \end{aligned} \tag{5}$$

where $\mathbf{w}$ stands for the reference signal vector. The reference model parameters should be chosen to fulfil the following equation:

$$\mathbf{C}_r(\mathbf{I}-\mathbf{A}_r)^{-1}\mathbf{B}_r=\mathbf{I} \tag{6}$$

which results in a unity gain for each channel. This enables reference trajectory tracking. For simplicity, we assume a first-order reference model. In such a case, matrices $\mathbf{A}_r$, $\mathbf{B}_r$, and $\mathbf{C}_r$ become diagonal. Furthermore, $\mathbf{C}_r$ can be chosen to be $\mathbf{I}$. To assure reference following at steady state, $\mathbf{B}_r$ has to be equal to $\mathbf{I}-\mathbf{A}_r$. The predictions of the reference trajectories are then written in the following form:

$$\mathbf{y}_r(k+H)=\mathbf{A}_r^H\mathbf{y}_r(k)+(\mathbf{I}-\mathbf{A}_r^H)\mathbf{w}(k) \tag{7}$$

with $\mathbf{A}_r$ given by

$$\mathbf{A}_r=\begin{bmatrix} a_{r_1} & 0 & \cdots & 0 \\ 0 & a_{r_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{r_m} \end{bmatrix} \tag{8}$$

and where a constant and bounded reference signal $[\mathbf{w}(k+i)=\mathbf{w}(k),\ i=1,...,H]$ is assumed. The main goal of the proposed algorithm is to find a control law that enables the *undelayed* controlled signal $\mathbf{y}_p^0(k)$ to track the reference trajectory.

To develop the control law, Eq. (7) is first rewritten in

$$\mathbf{w}(k+H)-\mathbf{y}_r(k+H)=\mathbf{A}_r^H[\mathbf{w}(k)-\mathbf{y}_r(k)]. \tag{9}$$

Taking into account the main idea of the proposed control law, the reference trajectory tracking $[\mathbf{y}_r(k+i)=\mathbf{y}_p^0(k+i),\ i=1,...,H]$, is given by

$$\mathbf{y}_p^0(k+H)=\mathbf{w}(k+H)-\mathbf{A}_r^H[\mathbf{w}(k)-\mathbf{y}_p^0(k)]. \tag{10}$$

The idea of MPFC is introduced by the equivalence of the objective increment vector $\boldsymbol{\Delta}_p$ and the model output increment vector $\boldsymbol{\Delta}_m$, that is,

$$\boldsymbol{\Delta}_p=\boldsymbol{\Delta}_m. \tag{11}$$

The former is defined as the difference between the predicted reference signal vector $\mathbf{y}_r(k+H)$ and the actual output vector of the *undelayed* plant $\mathbf{y}_p^0(k)$,

$$\boldsymbol{\Delta}_p=\mathbf{y}_r(k+H)-\mathbf{y}_p^0(k). \tag{12}$$

Substituting Eq. (10) into Eq. (12) yields

$$\begin{aligned}\boldsymbol{\Delta}_p&=\mathbf{y}_p^0(k+H)-\mathbf{y}_p^0(k)\\ &=\mathbf{w}(k+H)-\mathbf{A}_r^H[\mathbf{w}(k)-\mathbf{y}_p^0(k)]-\mathbf{y}_p^0(k).\end{aligned} \tag{13}$$

The variable $\mathbf{y}_p^0(k)$ cannot be measured directly. Rather, it will be estimated from the available signals:

$$\mathbf{y}_p^0(k)=\mathbf{y}_p(k)-\mathbf{y}_m(k)+\mathbf{y}_m^0(k). \tag{14}$$

It can be seen from Eq. (11) that the delay in the plant is compensated by the difference between the outputs of the *undelayed* and the *delayed* model. When a perfect model of the plant is available ($\mathbf{G}_m=\mathbf{G}_p$), the first two terms on the right side of Eq. (14) cancel and the result is actually the output of the *undelayed* plant. If this is not the case, only an approximation is obtained. The model output increment vector $\boldsymbol{\Delta}_m$ is defined by the following formula:

$$\boldsymbol{\Delta}_m=\mathbf{y}_m^0(k+H)-\mathbf{y}_m^0(k). \tag{15}$$

By substituting Eqs. (13) and (15) into Eq. (11) and making use of Eqs. (10) and (4) the following control law can be obtained:

$$\begin{aligned}\mathbf{u}(k)=\mathbf{G}_0^{-1}\{&(\mathbf{I}-\mathbf{A}_r^H)[\mathbf{w}(k)-\mathbf{y}_p^0(k)]+\mathbf{y}_m^0(k)\\ &-\mathbf{C}_m\mathbf{A}_m^H\mathbf{x}_m(k)\},\end{aligned} \tag{16}$$

where

$$\mathbf{G}_0=\mathbf{C}_m(\mathbf{A}_m^H-\mathbf{I})(\mathbf{A}_m-\mathbf{I})^{-1}\mathbf{B}_m. \tag{17}$$

The MPFC control law in analytical form is finally obtained by substituting Eq. (14) into Eq. (16),

$$\begin{aligned}\mathbf{u}(k)=\mathbf{G}_0^{-1}\{&(\mathbf{I}-\mathbf{A}_r^H)[\mathbf{w}(k)-\mathbf{y}_p(k)+\mathbf{y}_m(k)]\\ &+(\mathbf{A}_r^H\mathbf{C}_m-\mathbf{C}_m\mathbf{A}_m^H)\mathbf{x}_m(k)\}.\end{aligned} \tag{18}$$

Note that the control law (18) is realizable if $\mathbf{G}_0$ is nonsingular. This condition is true if the plant is stable, controllable, and observable. This means that the MPFC control law can be implemented only for the open-loop stable systems.

### *2.1. Properties of MPFC control law—Integral action*

The integral nature of the proposed algorithm will be shown, as follows. By using the relationship in Eq. (14), the control law in Eq. (18) becomes

$$\mathbf{u}(k)=\mathbf{G}_0^{-1}\{(\mathbf{I}-\mathbf{A}_r^H)[\mathbf{w}(k)-\mathbf{y}_p^0(k)+\mathbf{y}_m^0(k)] + (\mathbf{A}_r^H\mathbf{C}_m-\mathbf{C}_m\mathbf{A}_m^H)\mathbf{x}_m(k)\}. \tag{19}$$

The above can be transformed into a transfer function matrix form. By Eq. (3) the following equation is obtained:

$$\mathbf{U}(z)=\mathbf{G}_0^{-1}\{(\mathbf{I}-\mathbf{A}_r^H)[\mathbf{W}(z)-\mathbf{Y}_p^0(z)+\mathbf{Y}_m^0(z)]\} + \mathbf{G}_0^{-1}(\mathbf{A}_r^H\mathbf{C}_m-\mathbf{C}_m\mathbf{A}_m^H) \times(z\mathbf{I}-\mathbf{A}_m)^{-1}\mathbf{B}_m\mathbf{U}(z). \tag{20}$$

Solving the above equation for $U(z)$ yields

$$\mathbf{U}(z)=[\mathbf{G}_0-\mathbf{C}_m(\mathbf{I}-\mathbf{A}_m^H)(z\mathbf{I}-\mathbf{A}_m)^{-1}\mathbf{B}_m]^{-1} \times(\mathbf{I}-\mathbf{A}_r^H)[\mathbf{W}(z)-\mathbf{Y}_p^0(z)]. \tag{21}$$

It can be seen that the second term $[W(z) - Y_p^0(z)]$ represents the control error. The first term represents a controller in the feedforward path given by

$$\mathbf{G}_c(z)=[\mathbf{G}_0-\mathbf{C}_m(\mathbf{I}-\mathbf{A}_m^H)(z\mathbf{I}-\mathbf{A}_m)^{-1}\mathbf{B}_m]^{-1} \times(\mathbf{I}-\mathbf{A}_r^H). \tag{22}$$

Using Eq. (17), it is obvious that the matrix being inverted in Eq. (22) becomes $\mathbf{0}$ if $z=1$, which proves that the control law provides an integrating effect.

### *2.2. Properties of MPFC control law—Stability*

To study the stability properties of the multivariable predictive functional control, the following assumption is used. The study is limited to linear, time-invariant systems, assuming that a perfect model of the plant is available $[\mathbf{G}_m=\mathbf{G}_p,\ \mathbf{y}_p(k) =\mathbf{y}_m(k)]$ and that there is no external input to the closed-loop system ($\mathbf{w}=\mathbf{0}$). The multivariable predictive control law from Eq. (18) may than be written as

$$\mathbf{u}(k)=\mathbf{G}_0^{-1}(\mathbf{A}_r^H\mathbf{C}_m-\mathbf{C}_m\mathbf{A}_m^H)\mathbf{x}_m(k). \tag{23}$$

Introducing the control law from Eq. (23) into Eq. (3), the following is obtained:

$$\mathbf{x}_m(k+1)=[\mathbf{A}_m+\mathbf{B}_m\mathbf{G}_0^{-1}(\mathbf{A}_r^H\mathbf{C}_m -\mathbf{C}_m\mathbf{A}_m^H)]\mathbf{x}_m(k). \tag{24}$$

The closed-loop state transition matrix is defined as

$$\mathbf{A}_c=\mathbf{A}_m+\mathbf{B}_m\mathbf{G}_0^{-1}(\mathbf{A}_r^H\mathbf{C}_m-\mathbf{C}_m\mathbf{A}_m^H). \tag{25}$$

If Eq. (17) is substituted into Eq. (25) the following is obtained:

$$\mathbf{A}_c=\mathbf{A}_m+\mathbf{B}_m[\mathbf{C}_m(\mathbf{A}_m^H-\mathbf{I}) \times(\mathbf{A}_m-\mathbf{I})^{-1}\mathbf{B}_m]^{-1}(\mathbf{A}_r^H\mathbf{C}_m-\mathbf{C}_m\mathbf{A}_m^H). \tag{26}$$

To study the stability of the predictive controller, a stable open-loop plant with Hurwitz matrix $\mathbf{A}_m$ and Hurwitz reference model system matrix $\mathbf{A}_r$ are assumed.

**Remark 1**. *If the coincidence horizon is less than the maximal relative degree $\rho$ of the model ($H<\rho$) the matrix $\mathbf{G}_0$ becomes singular and the control law is not defined.*

**Remark 2**. *When the coincidence horizon is equal to the maximal relative degree of the model $H=\rho$, then the closed-loop poles tends to open-loop zeros $(n-m\cdot\rho)$ and the rest of the $(m\cdot\rho)$ poles are the solution of the following equation*:

$$|z^\rho\mathbf{I}-\mathbf{A}_r^\rho|=0. \tag{27}$$

*When the zeros of the open-loop system are not in the unit circle, the obtained closed-loop control is unstable*.

**Remark 3**. *When the coincidence horizon tends to infinity $(H\to\infty)$ the system matrix of the closed-loop system goes to $\mathbf{A}_m$:*

$$\lim_{H\to\infty}\mathbf{A}_c=\mathbf{A}_m. \tag{28}$$

From the remark above it can be concluded that a stable control law can always be obtained for open-loop Hurwitz systems $\mathbf{A}_m$, when a suitable coincidence horizon is used. The most suitable case is when the relative degree of the model is equal to 1 and perfect tracking can be obtained for $H=\rho$.

## 3. Model of the plant

The observed plant consists of a ventilator that conveys air, which serves as a transport medium, through the glass tube to the thermal insulated mixing chamber. The heating coil is mounted in the tube and the nebulizer is introduced downstream. The nebulizer is operated by compressed

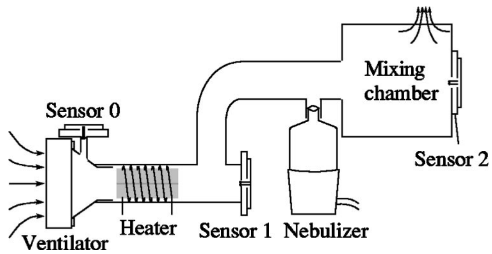


Fig. 1. Air conditioning pilot plant.

air provided by a small compressor. A mixture of heated and humidified air enters the mixing chamber as shown schematically in Fig. 1.

The plant has a square structure—with two inputs and two outputs. The controlled variables are the air temperature and relative humidity. These are measured with pairs of sensors at three points in the path of the air flow: at the air inlet to the plant, at a point between the heater and the nebulizer, and next to the air outlet in the mixing chamber. Corresponding control variables are the voltage of the heater and the voltage applied to the compressor motor. The operating point was chosen as the temperature, $\theta_2 = 22\,^\circ\mathrm{C}$ and the humidity, $\Phi_2 = 0.3$. Around this operating point the plant can be sufficiently modelled by the following discrete transfer function matrix:

$$\mathbf{G}_m(z) = \begin{bmatrix} \dfrac{0.004\,463 z^{-5}}{z-0.9835} & \dfrac{-0.000\,870\,4 z^{-2}}{z-0.9753} \\ \dfrac{-0.001\,895 z^{-30}}{z-0.995} & \dfrac{0.000\,534\,2 z^{-8}}{z-0.9876} \end{bmatrix}, \tag{29}$$

where $G_{m_{11}}(z)$ is the $Z$ transform of the output temperature $\Theta_2$ and the voltage of the heater $u_h$, $G_{m_{12}}(z)$ is the $Z$ transform of the output temperature $\Theta_2$ and the voltage applied to the compressor motor $u_v$, $G_{m_{21}}(z)$ is the $Z$ transform of the output humidity $\Phi_2$ and the voltage $u_h$ applied to the heater, and $G_{m_{22}}(z)$ is the $Z$ transform of the output humidity $\Phi_2$ and the voltage applied to the compressor motor $u_v$. The sampling time $T_s$ is 1 sec.

## 4. Comparison between Edmunds and MPFC method

The model of the plant given in Eq. (29) is used in a simulation study, where different multivariable control strategies are investigated and compared to proposed approach. The best performance and robustness among the classical methods are obtained using Edmunds' method [16], where the parameters of the multivariable controller are obtained by optimization. The tuning parameters in this case are a reference trajectory time constant of $T_{ref} = 40\,\mathrm{s}$ and a closed-loop frequency band of $\omega = [10^{-2}, 10^4]$. In the case of the MPFC controller, the reference trajectory is given by Eq. (30) with a coincidence horizon of $H = 40$. A comparison of both methods is shown in Fig. 2, where the responses of the first output of the multivariable process (temperature $\Theta_2$) of both control approaches are shown. In Fig. 3, a comparison of the second output (humidity $\Phi_2$) is presented and in Figs. 4 and 5, the corresponding control signals are shown. A comparison of both methods shows some advantages of the proposed approach. The method suppresses the interaction much better than in the case using Edmunds approach. However, in the case of reference tracking, both responses are similar. It should be emphasized that Edmunds method gave the best results amongst all classical methods including Maciejowski's method.

## 5. MPFC control design and implementation on the plant

The MPFC control algorithm has been implemented in real time on the air conditioning plant. The control of air conditioning plants is very important in the case of so-called white rooms in microelectronics, pharmacy, and biochemistry. In those cases the temperature and humidity must be controlled within very narrow tolerance bands. It is therefore very important to have a robust, high performance control system to ensure those conditions.

As compared to conventional techniques, the MPFC method, has the advantage of simplicity of design. After the modeling procedure, which is common for both predictive and conventional multivariable designs, the design of the multivariable predictive control becomes straightforward.

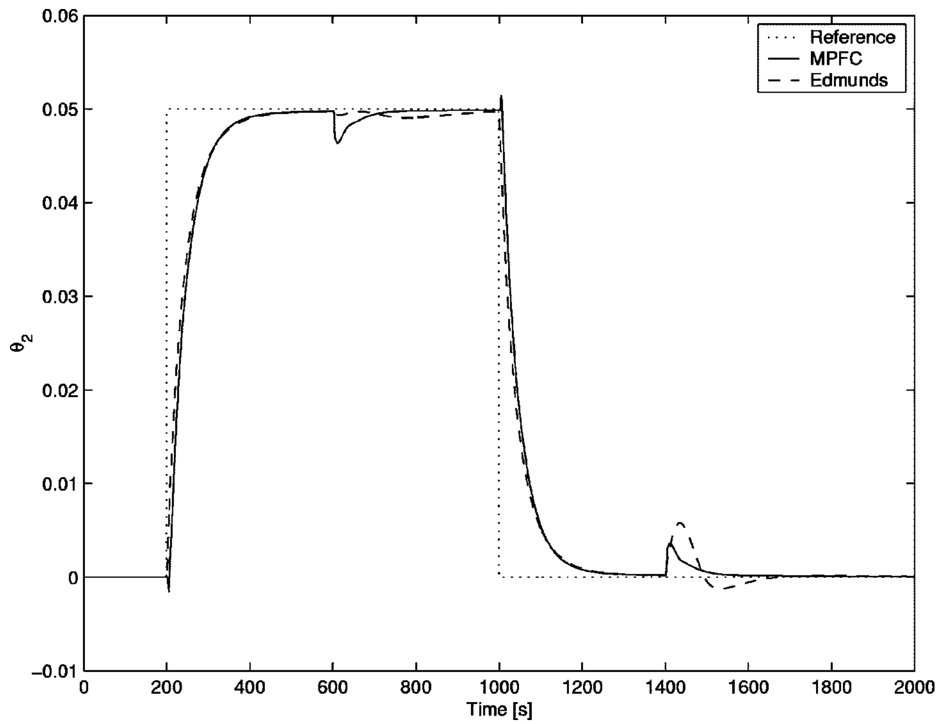


Fig. 2. A comparison between Edmunds approach and the MPFC controller—$\Theta_2$: temperature.

The main goal is to test the functionality of the proposed multivariable predictive functional control around a certain operating point. According, the continuous model is first obtained for a chosen operating point [Eq. (29)]. After discretization, the *delayed* model is obtained [Eq. (2)]. The *undelayed* model is transformed to a discrete state-space domain description (3) for design purposes. The design of the multivariable predictive controller is based on prescribed closed-loop dynamics that are defined in the form of the reference model in the state-space domain. In the present case, the reference model is chosen to be

$$\mathbf{A}_r=\begin{bmatrix} 0.9753 & 0 \\ 0 & 0.9753 \end{bmatrix}. \tag{30}$$

Faster responses are not realizable because of the input signal's constraints. The coincidence horizon $H$ influences the dynamics of the system response. Actually, it is the main influence on the proportional gain of the multivariable predictive controller. In the case of poor signal-to-noise ratio this parameter should be chosen carefully. A smaller value of the coincidence horizon results in a higher proportional gain, which also affects the noise propagation through the loop. In the experiment, $H=40$ is chosen to be 40.

In the experiment, both reference tracking and disturbance rejection are tested. This is achieved as follows: At the time 200 s, a change in the humidity reference is made; at time 600 s, a change in the temperature reference is made; at the time 1000 s, a change in the humidity reference is made; at time 1400 s, a change in the temperature reference is made; and, last, at time 1700 s, disturbance rejection is tested by changing the air flow through the glass tube.

The results of the real-time experiment are shown in Fig. 6 where the response of the controlled signals, the temperature $\vartheta_2$ and the humidity $\varphi_2$ are shown, and in Fig. 7 where the corresponding control signals, the voltage $u_h$ applied to the heater and the voltage $u_v$ applied to the compressor motor, are presented.

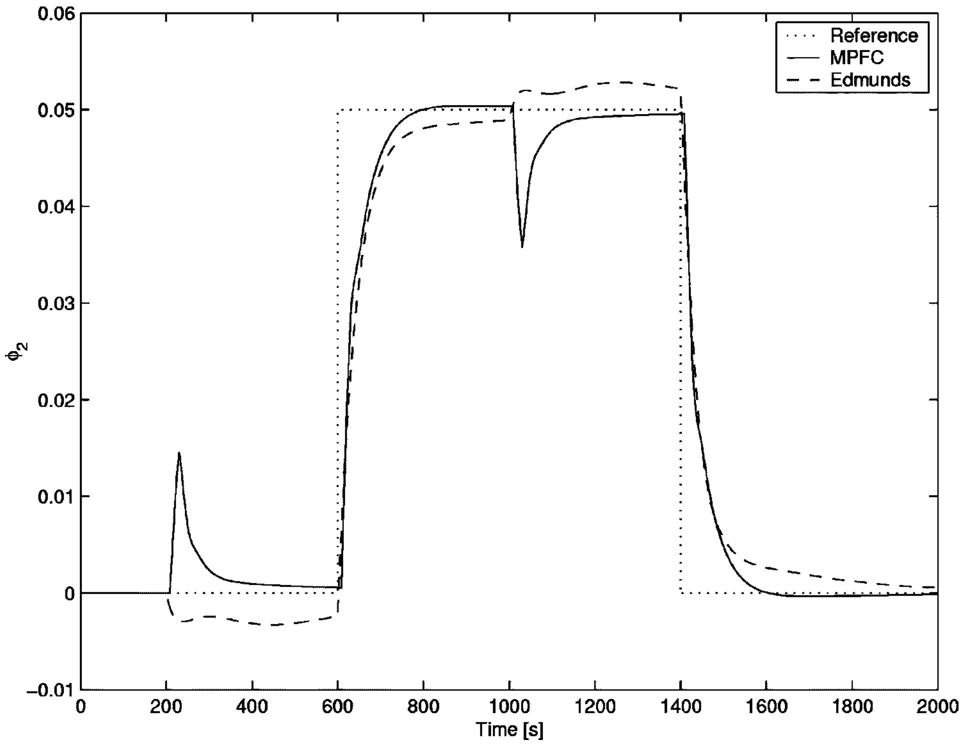


Fig. 3. A comparison between Edmunds approach and the MPFC controller—$\Phi_2$: humidity.

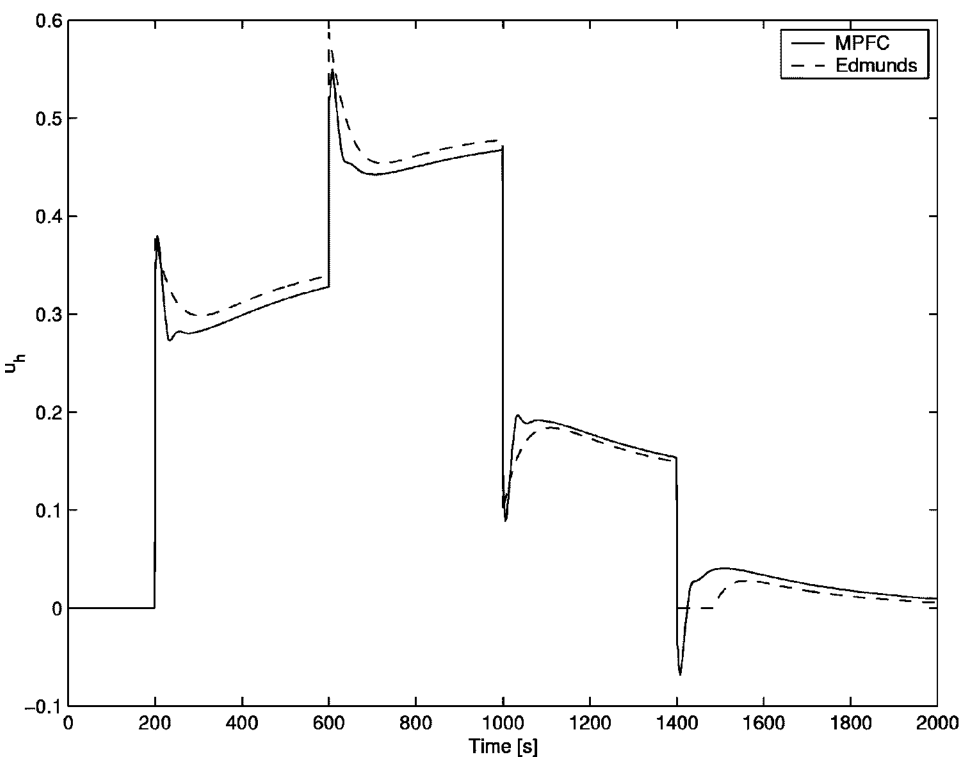


Fig. 4. A comparison between Edmunds approach and the MPFC controller—$u_h$: manipulated variable.

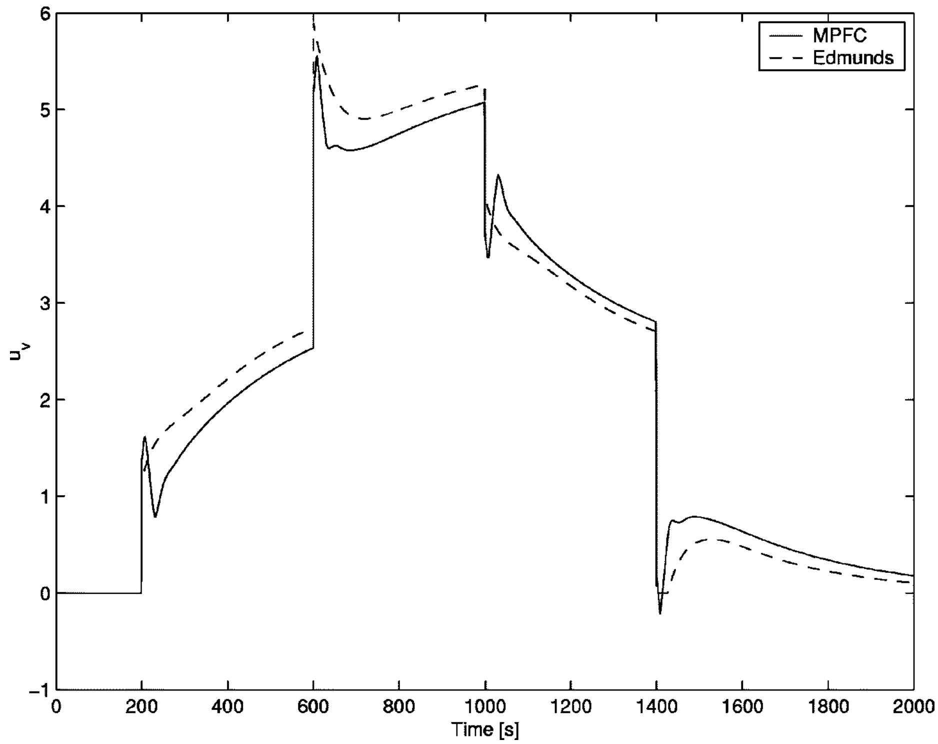


Fig. 5. A comparison between Edmunds approach and the MPFC controller—$u_v$: manipulated variable.

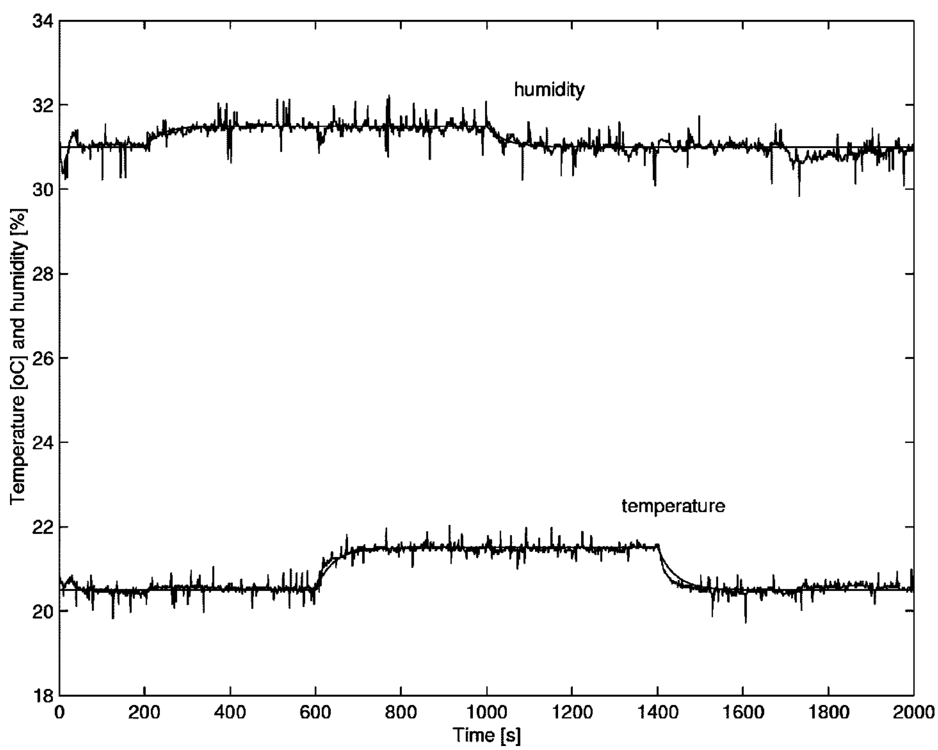


Fig. 6. Multivariable predictive functional control of an air conditioning system: temperature and humidity.

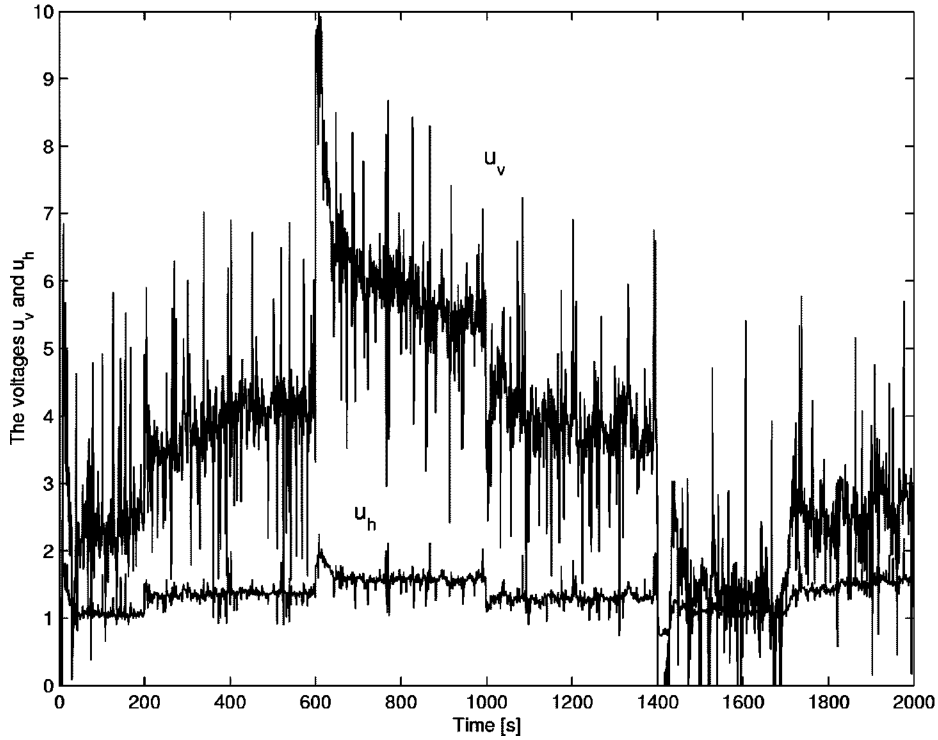


Fig. 7. Multivariable predictive functional control of an air conditioning system: manipulated variables.

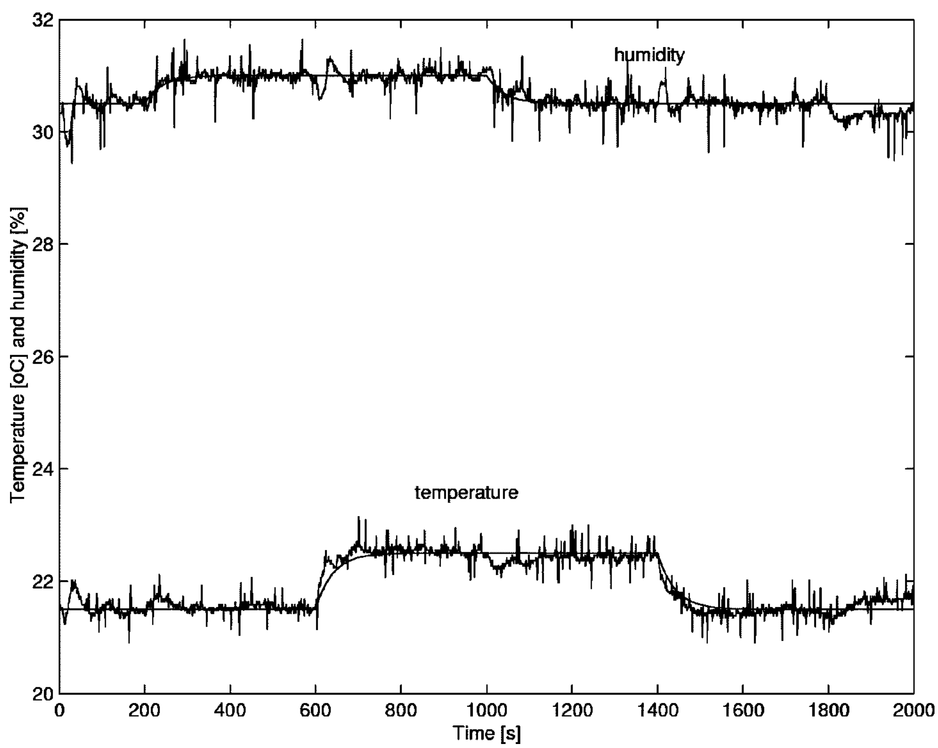


Fig. 8. Multivariable predictive functional control of an air conditioning system with filter: temperature and humidity.

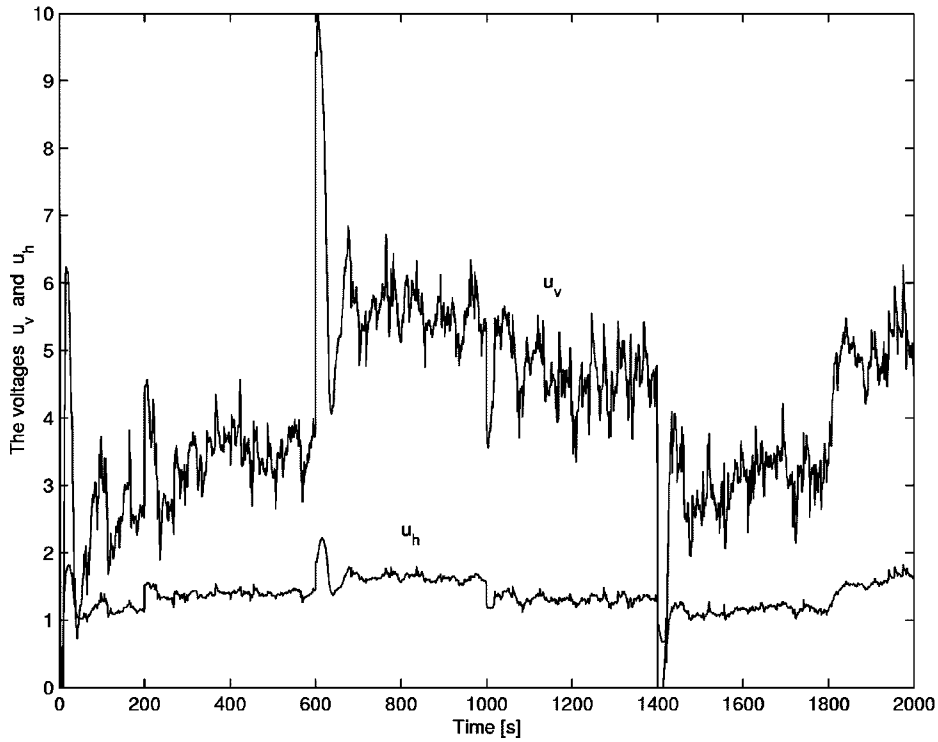


Fig. 9. Multivariable predictive functional control of an air conditioning system with filter: manipulated variables.

As shown in Fig. 6, both reference tracking and disturbance rejection modes of the multivariable predictive functional control exhibit reasonable performances. The problem that arises is poor quality (noise) of the humidity sensor. This means that the voltage $u_v$ is affected by the present of the noise.

Since MPFC is a form of internal model control, a filter is applied to improve the behavior of the control law in the presence of noise. The discrete version of the filter is given by

$$F(s) = \frac{1-\lambda}{(z-\lambda)^n}. \tag{31}$$

To obtain a simple solution [17], the filter order is chosen to be 1. The results are shown in Figs. 8 and 9. The former shows the controlled signals and the latter the manipulated variables. It is obvious that the actuators are less affected by the noise and thus the performance of the system is satisfactory.

## 6. Conclusion

In the paper a new multivariable predictive control has been presented. It is based on the principles of predictive functional control, a method which has found numerous successful applications in industry. The main advantage of the proposed algorithm is in the simple design even in the case of delayed systems. The proposed algorithm was tested on a multivariable air condition plant. In addition to having time delays, the plant had poor measurement signal due to high frequency noise. Nevertheless, the proposed MPFC gave satisfactory closed-loop results when tested on the real air conditioning plant.

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**Igor Škrjanc** was born in Ljubljana, Slovenia, in 1963. He received his engineering diploma from the Faculty of Electrical Engineering, Ljubljana in 1988, a Master's degree in Electrical Engineering from the University of Ljubljana in 1991, and a doctorate from the same university in 1996. His dissertation is on adaptive fuzzy systems. He is currently Associate Professor at the same faculty. His main research interests are in adaptive, predictive, fuzzy, and fuzzy adaptive control systems.

**Sašo Blažič** received the B.Sc., M.Sc., and Ph.D. degrees in 1996, 1999, and 2002, respectively, from the Faculty of Electrical Engineering, University of Ljubljana. Currently, he is Assistant Professor at the same faculty. His main interests include adaptive and fuzzy control with stress on the robustness issues of adaptive control.

**Simon Oblak** was born in 1979 in Ljubljana, Slovenia. He received his B.Sc. degree in Electrical Engineering from the University of Ljubljana in 2003. He is currently working at the Laboratory of Process Automation and Informatisation. His main interests are multivariable predictive control and fuzzy identification.

**Jacques Richalet** was born in Versailles, France, in 1936. He received his engineering diploma from the Ecole Nationale Superieure de l'Aeronautique in 1960, a Master's degree in Electrical Engineering from University of California (Berkeley) in 1961, and a doctorate in Applied Mathematics in 1965 from the University of Paris. He has been active in predictive control and in process control applications for several years. Since 1971 he has been scientific director of Adersa which is engaged in modeling and control of industrial processes.